Journal of Engineering Physics and Thermophysics, Vol. 68, No. 2, 1995

ESTIMATION OF THE TWO-PHASE FLOW ON A HORIZONTAL SMOOTH PLATE IN THE LAMINAR AND TRANSIENT REGIONS

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The flow of thin liquid films set in motion by a vapor flow over a smooth horizontal surface has been estimated. Shear stresses are calculated by a gridpoint method for laminar smooth, laminar wavy, and transient regions of the flow. The calculations are compared with experimental measurements of shear stresses on the plate. Reasonable agreement between the calculations and the experiments has shown that the method suggested can be used for solution of a wide range of applied problems.

Two-phase working fluids are widely used [1, 2] in power units, apparatus of chemical engineering, and various kinds of facilities of advanced technology. Two-phase flows of working fluids in continuous-flow sections of heat exchangers and components of engines are accompanied by settling of thin liquid films on surfaces in the flow [2]. Attempts to reduce the size of apparatus and engines have led to increases in the velocities of the vapor component of the flows. The presence of the liquid film on shaped surfaces and elements of continuous-flow sections, most frequently calculated from equations of single-phase gas dynamics, is a source of additional losses of energy by the working fluid and it changes the conditions of heat and mass transfer and the flow pattern over the shaped surfaces.

A large number of works, which have been surveyed in [1–12], concern flow of smooth and wavy thin liquid films that are set in motion by a high-velocity vapor flow or that run down over solid surfaces under the action of gravity. Studies of sea waves have contributed much to investigation of wave motions on a water surface. However, extension of experimental and theoretical findings obtained for sea waves to estimation of wave processes on the surface of thin films covered by capillary waves is inadequate and gives substantial errors.

In the single-phase gas dynamics of turbomachines simulation by "sand" roughness is widely used for estimation of losses due to roughness of surfaces in the flow. However, according to [7], substitution of a "sand" or corrugated rigid surface for the wave spectrum [5] results in approximately twofold overestimation of velocities in the wall layer. Therefore, this substitution can be valid only for estimation in a first approximation.

Survey [8] is devoted to mathematical modeling of two-phase flows in channels and boundary layers with consideration of various models of two-phase mixtures. A detailed relevant biography is given there.

As far as the number of theoretical and experimental publications, motion of smooth and wavy films is one of the most thoroughly developed branches of two-phase fluid mechanics. However, the observed great variety of wave flow patterns in the surface layer of films and different patterns of capillary wave motions [8, 10] continue to attract attention to theoretical and experimental investigation of the motion of liquid films and gas flows over wavy surfaces [3-6].

Calculation of blade passages, in particular, in wet-steam (WS) turbines working with two-phase flows and turbomachines with liquid film cooling, requires representative estimation of velocities of the steam components of the flow over the moisture film on shaped surfaces of a passage. Recent studies [9] have demonstrated the need for special rational shaping of the surfaces of nozzle vanes and rotor blades of WS or two-phase liquid-metal turbine stages. Experimental and theoretical techniques have been developed for rational shaping of turbine blade passages

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St. Petersburg University, St. Petersburg, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 68, No. 2, pp. 184-191, March-April, 1995. Original article submitted March 19, 1991; revision submitted May 19, 1992.

[7, 9]. Estimation of the initial conditions of two-phase flows with a moisture content up to 8–15% requires substantiated calculation of a two-phase film-droplet wall layer and determination of the velocity of the steam component on the external boundary of the flowing film. The need to use experimental quantities, for example, the coefficients of friction of the steam flow over the wavy film, is caused by a large number of factors that affect the steam-film flow in the wall layer.

For estimation of the shear stresses of the steam flow on the surface of a laminar liquid film, calculations were carried out using differential equations of motion of a boundary layer over a smooth horizontal surface [11]:

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = - \frac{\partial p_1}{\partial x} + \frac{\mu_1}{\rho_1} \frac{\partial^2 u_1}{\partial y^2};$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0; \tag{1}$$

$$\frac{\rho_2 \mu_2}{\rho_1 \mu_1} \left(u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y} \right) = - \frac{\mu_2}{\mu_1} \frac{\partial p_2}{\partial x} + \frac{\mu_2}{\mu_1} \frac{\partial^2 u_2}{\partial y^2};$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0.$$

The boundary conditions on the wall and external boundary of the boundary layer are

$$u_2 = v_2 = 0 \quad \text{at} \quad y = 0; \quad u_1 \to U_1 \quad \text{at} \quad y \to \infty. \tag{2}$$

On the phase interface – the line of discontinuity of physical properties of the medium – relations obtained from the laws of conservation of momentum and mass should be satisfied. In calculations of the flow of the two-phase boundary layer by Eqs. (1) with boundary conditions (2), following [7, 9, 11], the following assumptions were used: the flow of the smooth film is steady; for the range $\mathrm{Re_{fm}} = 0–80$ the film can be assumed to be smooth in a first approximatiion; the flow of the phases occurs without heat transfer, phase transitions, or mass transfer through the phase interface.

The condition of permeability of the phases has the form: $v_2 - u_2 d\delta_2/dx = v_1 - u_1 d\delta_2/dx = 0$. The momentum equation in the projection onto the normal the $n-n$ to the interface is $p_1 = p_2$. The momentum equation in the projection onto the tangent τ to the interface is $\mu_1 \partial u_1/\partial y = \mu_2 \partial u_2/\partial y$. Absence of slip on the interface $u_1 = u_2$ is adopted as an additional condition.

The conventional model of interaction of the phases stipulates that the effect on the averaged characteristics of the film be exerted in a single way, namely, through the friction stress τ on the interface. The adopted model seems to be the simplest. It can be reduced to viscous interaction of two boundary layers with different viscosities and densities.

The nonlinear system of equations (1) has been solved by the gridpoint method with approximation in the form of an implicit six-point scheme [12] on the main integral rectangular and semi-integral grid. The continuity equations are approximated by a four-point grid. A parabolic velocity distribution was assumed initially for the film and an "impact" velocity profile was assumed for the steam. A velocity profile close to a linear one was arrived at across the film by the method of successive approximations with boundary conditions (2). The shear stresses across the film remained constant.

The suitability of the model adopted for the steam-film system in the range of Reynolds numbers indicated was verified using experimental data of [7] (Fig. 1, curves 1–3). In the calculation $\mathrm{Re_{fm}} \in 40–200$ and Re_{xs} of the steam flow over the film were varied by varying the flow rate in the film and the Mach number M_1. Reasonable agreement of calculations using $\bar{\tau} = \tau_c/\tau_{0c}$ is observed in the range of $\mathrm{Re_{fm}}$ up to 80–85. A further increase in $\mathrm{Re_{fm}}$ leads to divergence of experiments and calculations, and the model assumed becomes invalid.

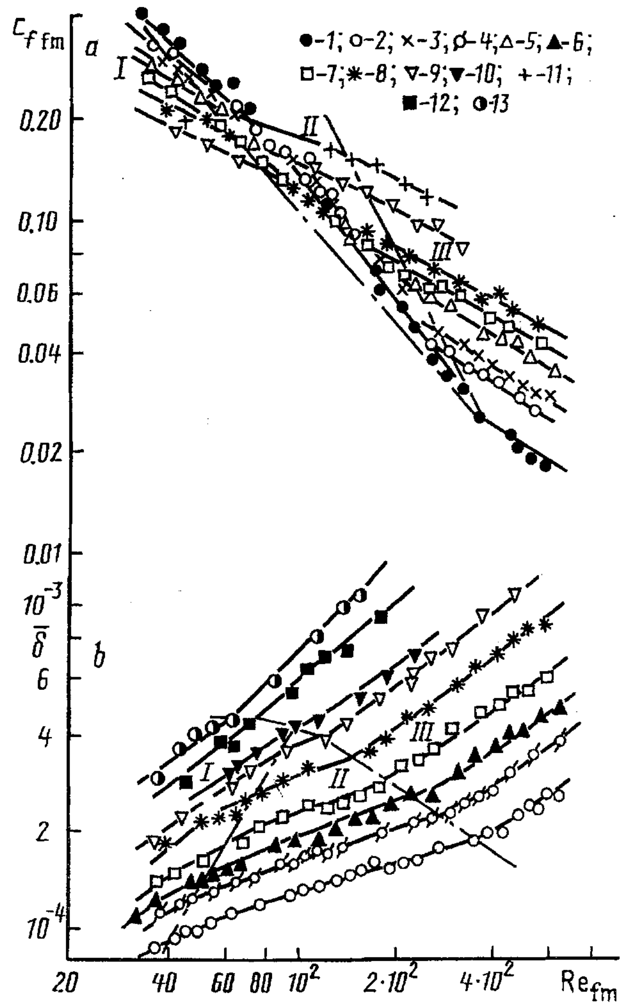

Fig. 1. Plot of the local friction coefficient (a) and the average thickness of the film (b) versus the Reynolds numbers of the film and the steam: $Re_{xs}\cdot 10^{-5}$: 1) 11.2, 2) 8.5, 3) 7.4, 4) 6.5, 5) 5.8, 6) 5.7, 7) 4.5, 8) 3.4, 9) 2.6, 10) 2.4, 11) 2.1, 12) 1.9, 13) 1.6; I, II, III) laminar, transitional, and turbulent regions of the film flow. $\overline{\delta}$, m.

Analysis of experimental data [7, 9] on the behavior of the local friction coefficient on the interface as a function of Re_{fm} and Re_{xs}, estimated by measuring the friction stress on the wall in a nongradient flow (Fig. 2), has revealed the following characteristic regions for Re_{fm}: I, $Re_{fm} < 80-100$; II, $40-100 < Re_{fm} < 380$; III, for various Re_{xs}, $100 < Re_{fm} < 400$. These regions characterize laminar, transitional, and turbulent flows in the film, respectively. Because of instability of the right-hand outer boundary of the transitional region, it is traced implicitly.

For the majority of practical problems, including calculation of a two-phase flow (of wet steam) in passages of turbine stages, transitional and turbulent flows (II and III) are of definite interest. We will use the conventional model of phase interaction [9], where a liquid film flows over the surface of a passage under the action of steam. The film flow will be characterized by the dimensionless quantities $\overline{u}_2^+ = u_2/v_*$, $y^+ = y\rho_2 v_*/\mu_2$, and $\delta^+ = \overline{\delta}\rho_2 v_*/\mu_2$ and the dynamic velocity $v_* = (\tau_0/\rho_2)^{0.5}$.

A two-layer model of turbulent viscosity will be taken for the transitional and initial turbulent two-phase flows. In the region of the viscous sublayer and the transitional zone ($0 \le y^+ \le 20$) we will use the semiempirical equation [15] $\mu_{T2}/\mu_2 = n_2^2\overline{u}^+ y^+ [1 - \exp(-n_2^2\overline{u}_2^+ y^+)]$, where $n_2 = f(Re_{xs}, Re_{fm})$ is an empirical constant. For the turbulent region of the film flow, located over the viscous sublayer, we will use the Prandtl equation $\overline{\mu}_{T2}/\mu_2 = \kappa_2^2 y^{+2} d\overline{u}_2^+/dy^+$, where $\kappa_2 \cong 0.4$ is the turbulence constant.

The dynamic velocity v_* is taken as the scale of fluctuation velocities for the steam part of the wall layer. The change in the turbulence scale ε was determined from the adopted logarithmic profile of the velocity of the

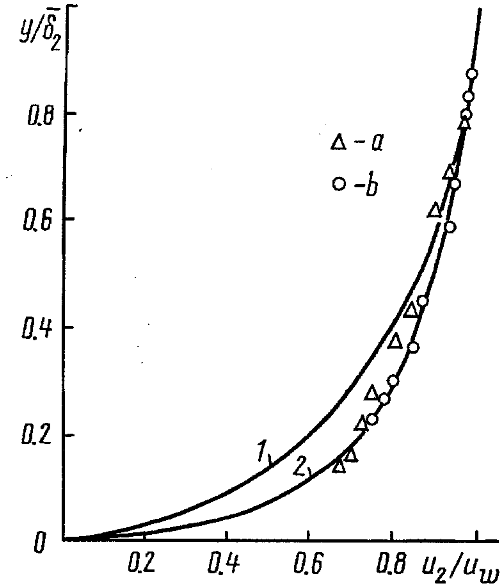

Fig. 2. Velocity profiles in the film versus Reynolds numbers of the film and the steam [7]: a) $\mathrm{Re_{fm}} = 187$, $\mathrm{Re_{xs}} = 10.5 \cdot 10^5$; b) $\mathrm{Re_{fm}} = 195$, $\mathrm{Re_{xs}} = 1.9 \cdot 10^5$; 1, 2) calculation: 1) $\mathrm{Re_{xs}} = 9.01 \cdot 10^5$, 2) $2.1 \cdot 10^5$.

steam over the film [9]. The ratio of the local friction stress τ_1 to the friction stress on the interface τ_w was found in the form of the polynomial [13]

$$\tau_1/\tau_w = a + a'(y_1/\delta_1) + a''(y_1/\delta_1)^2 . \tag{3}$$

The coefficients of the polynomial were determined from the boundary conditions: $y_1 = 0$; $u_1 = u_w$; $v_1 = v_w$; $y_1 = \delta_1$; $u_1 = U$; $\tau_1 = 0$. Solution of Eq. (3) with the boundary conditions adopted gives the values of the coefficients of the polynomial: $a = 1$; $a' = (\delta_1\rho_1/\tau_w)[(u_1\partial u_1/\partial x)|_w + (v_1\partial u_1/\partial y)|_w]$; $a'' = -(a + a')$.

The total viscosity of the steam component of the flow $\tilde{\mu}_1$ was estimated by superposition of the molecular viscosity μ_1, the viscosity μ'_{T1} determined by the distance from the average surface of the film $\bar{\delta}_2$, and the turbulent viscosity μ''_{T1} determined by the wavy roughness of the surface of the liquid film [14]: $\tilde{\mu}_1 = \mu_1 + \mu'_{T1} + \mu''_{T1}$.

With account for the dimensionless characteristics u_1^+ and y_1^+, the expression for τ will be given in the form

$$\tau_1/\tau_w = (1 + \mu_{T1}/\mu_1)\,\partial u_1^+/\partial y_1^+ . \tag{4}$$

We will introduce a term modeling vortex formation due to the wavy surface of the film f_2 [15], and assuming no laminar sublayer over the film, we obtain

$$\mu_{T1}/\mu_1 = (\kappa_1 y_1^+ + f_2)\,\tau_1/\tau_w . \tag{5}$$

With the boundary conditions $y_1^+ = 0$ and $u_1^+ = u_w^+$, integration of Eq. (4) with (5) and neglect of the molecular viscosity μ_1 give

$$u_1^+ = \kappa_1^{-1} \ln (\kappa_1 y_1^+ + f_2) f_2^{-1} + u_w^+ . \tag{6}$$

Use will be made of the velocity profile (Fig. 2) [9] measured over the film flowing over the plate. For the transitional flow region over the film we will use the relation

$$u_1^+ = \kappa_1^{-1} \ln y_1^+ + D: \;\; D = f(\mathrm{Re_{xs}}, \mathrm{Re_{fm}}) . \tag{7}$$

At a distance from the film $\kappa_1 y_1^+ \gg f_2$. Neglecting f_2 compared to $\kappa_1 y_1^+$, f_2 will be estimated from Eqs. (6) and (7):

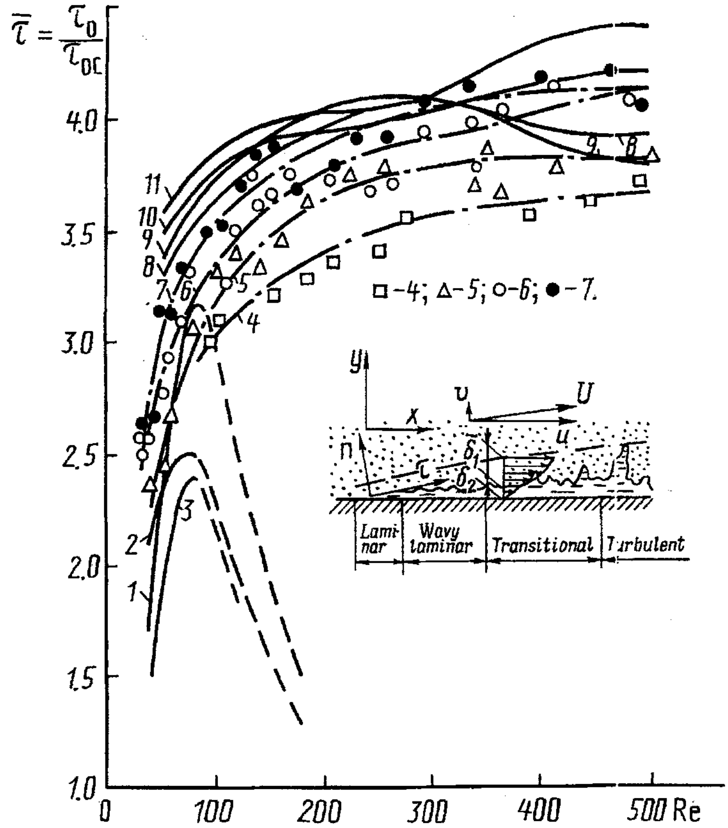

Fig. 3. Effect of the Reynolds number of the film and the steam flow on the friction of the steam on the film surface: 1–3) calculated from Eqs. (1): 1) $Re_{xs} = 6.19 \cdot 10^5$, $M_1 = 0.44$; 2) $Re_{xs} = 8.1 \cdot 10^5$, $M_1 = 0.63$; 3) $Re_{xs} = 9.2 \cdot 10^5$, $M_1 = 0.84$; 4–7) [7]: $Re_{xs} \cdot 10^5$: 4) 4.5; 5) 5.9; 6) 8.5; 7) 11.2; 8–11) calculated from Eqs. (10): 8) $Re_{xs} \cdot 10^{-5} = 9$; 9) 10; 10) 6; 11) $Re_{xs} \cdot 10^{-5} = 1.5$.

$$f_2 = \kappa_1 \exp\left[\kappa_1 \left(\overset{+}{u}_w - D\right)\right]. \tag{8}$$

Substituting Eq. (8) into Eq. (5), we obtain

$$\mu_{T1}/\mu_1 = \left\{ \kappa_1 \overset{+}{y}_1 + \kappa_1 \exp\left[\kappa_1 \left(\overset{+}{u}_w - D\right)\right] \right\} \tau_1/\tau_w. \tag{9}$$

Numerical solutiom of boundary layer equations (1) and (2) was carried out for the transitional and initial turbulent regions $Re_{fm} \in (40-80)-500$ with transformation of the coordinate system:

$$\text{for the steam} \quad \xi_1 = x;\ \eta_1 = \ln\left(1 + y_1/\delta_1 A_1\right);$$

$$\text{for the film} \quad \xi_2 = x;\ \eta_2 = \ln\left(1 + y_2/\delta_2 A_2\right).$$

With the new coordinate system it was possible to carry out calculations with a constant step and the same number of points in the transverse direction with a change in δ_1 and δ_2. The calculational equations for the steam ($i = 1$) and the film ($i = 2$) were represented in the form

$$\rho_i \left\{ \left(A_i \delta_i \exp \eta_i\right)^2 u_i \frac{\partial u_i}{\partial \xi_i} + \left[v_i A_i \delta_i \exp \eta_i - u_i A_i^2 \left(\exp \eta_i - 1\right) \times \right.\right.$$

$$\left.\left. \times \left(\exp \eta_i\right) \delta_i \frac{d\delta_i}{d\xi_i} + \frac{dh}{d\xi_i} + \frac{\tilde{\mu}_i}{\rho_i} \right] \frac{\partial u_i}{\partial \eta_i} \right\} = \frac{\partial}{\partial \eta_i} \left(\mu_i \frac{\partial u_i}{\partial \eta_i} \right); \tag{10}$$

$$A_i (\exp \eta_i)\, \delta_i \frac{\partial u_i}{\partial \xi_i} - \left[A_i (\exp \eta_i - 1) \frac{d\delta_i}{d\xi_i} + \frac{dh}{d\xi_i} \right] \frac{\partial u_i}{\partial \eta_i} + \frac{\partial v_i}{\partial \eta_i} = 0\,;$$

$i = 1$ for $y > \bar{\delta}_2$; $i = 2$ for $0 \le y \le \bar{\delta}_2$. For the film $h = 0$; for the steam $h = \bar{\delta}_2$.

The boundary conditions and the expression for $\tilde{\mu}$ were transformed accordingly. Equations (10) were solved by overall factorization. Lacking factorization coefficients were found from the condition $\tau_{w_1} = \tau_{w_2}$. After each iteration the thickness of the film was corrected for nonlinearity with the condition that the film flow rate was constant. The calculation method used allowed us to determine the thickness of the boundary layer δ_1 without using iteration and to obtain the sufficiently smooth function $\delta_1 = f(x)$.

The results calculated in the considered range of Re_{xs} and Re_{fm} were compared with experimental data of [9] (see Fig. 3). For $\mathrm{Re}_{fm} \in 80-350$ and $\mathrm{Re}_{xs} = 11.2 \cdot 10^5$, points 7 and calculated curve 9 agree quite well. Experimental data (points 6) and calculations (curve 8) also agree fairly well at $\mathrm{Re}_{xs} = (8.5-9) \cdot 10^5$.

As the thickness of the film increased [12], the amplitude of the capillary waves on the surface of the film and the surface roughness of the interface increased. This can probably explain the scatter of experimental points at $\mathrm{Re}_{fm} > 350$. At $\mathrm{Re}_{fm} > 350$, calculated curves 8 and 9 follow Eq. (10), which corresponds to the scatter of experimental points.

At lower Reynolds numbers $\mathrm{Re}_{xs} < (5-6) \cdot 10^5$ agreement of calculated and experimental data is not as good. As the Reynolds numbers Re_{xs} decrease, in the two-phase wall layer the length of transitional region II is shortened with increasing Re_{fm} (see Fig. 1b). The film thickness $\bar{\delta}_2$ increases and the average liquid flow rate in the film decreases. As Re_{xs} decreases and Re_{fm} increases, the absolute value of the local friction coefficient $c_{f\,fm}$ increases with tendency to decrease as Re_{fm} increases (Fig. 1a). From the aforesaid it is possible to suggest that the turbulent wave mechanism of friction on the interface of the film and the steam-droplet flow changed. Neglect of unsteady-state effects in the rough-film flow and some other hydrodynamic features, such as separation of droplets from the film, restructure of the capillary waves on the film, and changes in the dissipative component of the flow energy, leads to changes in the dependence of the total viscosity $\tilde{\mu}$ of the steam flow and its components that is adopted in the calculations. The present calculation results were affected by the adopted analogy in the form of a "wall law" [7] of a fixed roughness as applied to a mobile and variable wavy surface in transitional and turbulent regions II and III (Fig. 1). In order to take account of the indicated features of the flow of the wall layers in the estimations, it is necessary to carry out special experiments for transitional and turbulent regions of the film.

The present calculation method can be used for solution of a wide range of applied problems such as refinement of the boundary conditions on shaped surfaces of passages in turbomachines working with steam-gas-liquid flows, components of heat exchangers, ejectors, and other apparatus.

CONCLUSIONS

1. Estimation of parameters of a two-phase wall layer using suggested equations (10) gives agreement with experiments that is acceptable for engineering calculations [7, 9] at $\mathrm{Re}_{xs} = 6 \cdot 10^5 - 11 \cdot 10^5$ and $\mathrm{Re}_{fm} \in (40-80)-350$.

2. The proposed method for calculating wall boundary conditions can be used to solve a wide range of applied problems in shaping passages in turbomachines operating with two-phase working fluids, in calculating surfaces of heat exchangers coated with a flowing film, etc.

3. For Reynolds numbers of the steam component of a flow $\mathrm{Re}_{xs} < (5-6) \cdot 10^5$ suitable results can be obtained by the proposed method only after further experiments to determine the total viscosity $\tilde{\mu}$ of the two-phase wall layer.

NOTATION

x, y, ξ, η, coordinates; U, velocity of the steam flow; u, v, projections of the steam velocity vector $\mathbf{U}$, onto the x, y axes; p, pressure; ρ, density; μ, viscosity; $\tilde{\mu}$, total (effective) viscosity; μ_T, turbulent viscosity; τ, friction stress; $\bar{\tau}$, relative friction stress; κ, n, turbulence coefficients; ε, turbulence scale; f, dimensionless quantity modeling turbulization of the flow by a wavy roughness; D, experimental parameter; A, A_1, A_2, parameter, concentration parameters; $\bar{\delta}_2$, δ_2, average thickness, thickness of the film; δ_1, thickness of the boundary layer; M, Mach number; Re, Reynolds number; v_*, dynamic velocity; u^+, dimensionless velocity; y^+, dimensionless coordinate; in Eq. (10) $h = y - y_1$, for the film $h = 0$, for the steam $h = \delta_2$. Subscripts: for the steam $i = 1$; for the film $i = 2$; 0, value at the wall; w, value at the interface; $^-$, averaging symbol; fm, film; xs, steam, value in the x direction.

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